PHY1322B Winter 2017 STUDENT NAME:________________

ASSIGNMENT 9: Quantum Physics Bohr's Atom STDUENT'S ID________________

Due: Mar 29 6 PM

1 An isolated copper sphere of radius 5.00 cm, initially uncharged, is illuminated by ultraviolet light of wavelength 200 nm. What charge will the photoelectric effect induce on the sphere? The work function for copper is 4.70 eV.

Ultraviolet photons will be absorbed to knock electrons out of the sphere with maximum kinetic energy $K_{\max} = hf - \phi$,

or $$K_{\max} = \frac{(6.626\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})}{200\times10^{-9}\ \text{m}}\left(\frac{1.00\ \text{eV}}{1.60\times10^{-19}\ \text{J}}\right) - 4.70\ \text{eV} = 1.51\ \text{eV}.$$

The sphere is left with positive charge and so with positive potential relative to $V = 0$ at $r = \infty$. As its potential approaches 1.51 V, no further electrons will be able to escape, but will fall back onto the sphere. Its charge is then given by

$$V = \frac{k_e Q}{r} \quad \text{or} \quad Q = \frac{rV}{k_e} = \frac{(5.00\times10^{-2}\ \text{m})(1.51\ \text{N}\cdot\text{m/C})}{8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2} = \boxed{8.41\times10^{-12}\ \text{C}}.$$

2 A student studying the photoelectric effect from two different metals records the following information: (i) the stopping potential for photoelectrons released from metal 1 is 1.48 V larger than that for metal 2, and (ii) the threshold frequency for metal 1 is 40.0% smaller than that for metal 2. Determine the work function for each metal.

From condition (i), $hf = e(\Delta V_{S1}) + \phi_1$ and $hf = e(\Delta V_{S2}) + \phi_2$

$$(\Delta V_{S1}) = (\Delta V_{S2}) + 1.48\ \text{V}.$$

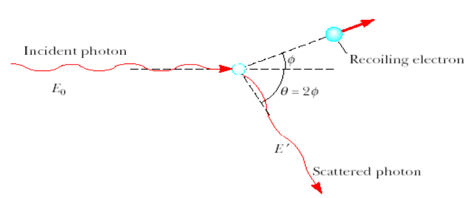
Then $\phi_2 - \phi_1 = 1.48\ \text{eV}$.

From condition (ii), $hf_{c1} = \phi_1 = 0.600hf_{c2} = 0.600\phi_2$

$$\phi_2 - 0.600\phi_2 = 1.48\ \text{eV}$$

$$\boxed{\phi_2 = 3.70\ \text{eV}} \quad \boxed{\phi_1 = 2.22\ \text{eV}}.$$

3 A 0.700-MeV photon scatters off a free electron such that the scattering angle of the photon is twice the scattering angle of the electron (Fig. P40.28). Determine (a) the scattering angle for the electron and (b) the final speed of the electron



(a) Thanks to Compton we have four equations in the unknowns ϕ, v, and λ':

$$\frac{hc}{\lambda_0} = \frac{hc}{\lambda'} + \gamma m_e c^2 - m_e c^2 \quad \text{(energy conservation)} \quad [1]$$

$$\frac{h}{\lambda_0} = \frac{h}{\lambda'}\cos 2\phi + \gamma m_e v \cos\phi \quad \text{(momentum in } x \text{ direction)} \quad [2]$$

$$0 = \frac{h}{\lambda'}\sin 2\phi - \gamma m_e v \sin\phi \quad \text{(momentum in } y \text{ direction)} \quad [3]$$

$$\lambda' - \lambda_0 = \frac{h}{m_e c}(1 - \cos 2\phi) \text{ (Compton equation).} \quad [4]$$

Using $\sin 2\phi = 2\sin\phi\cos\phi$ in Equation [3] gives $\gamma\, m_e v = \dfrac{2h}{\lambda'}\cos\phi$.

Substituting this into Equation [2] and using $\cos 2\phi = 2\cos^2\phi - 1$ yields

$$\frac{h}{\lambda_0} = \frac{h}{\lambda'}\left(2\cos^2\phi - 1\right) + \frac{2h}{\lambda'}\cos^2\phi = \frac{h}{\lambda'}\left(4\cos^2\phi - 1\right),$$

or $\lambda' = 4\lambda_0\cos^2\phi - \lambda_0$

. [5]

Substituting the last result into the Compton equation gives

$$4\lambda_0\cos^2\phi - 2\lambda_0 = \frac{h}{m_e c}\left[1 - \left(2\cos^2\phi - 1\right)\right] = 2\frac{hc}{m_e c^2}\left(1 - \cos^2\phi\right).$$

With the substitution $\lambda_0 = \dfrac{hc}{E_0}$, this reduces to

$$\cos^2\phi = \frac{m_e c^2 + E_0}{2m_e c^2 + E_0} = \frac{1+x}{2+x} \text{ where } x \equiv \frac{E_0}{m_e c^2}.$$

For $x = \dfrac{0.700\text{ MeV}}{0.511\text{ MeV}} = 1.37$, this gives $\phi = \cos^{-1}\sqrt{\dfrac{1+x}{2+x}} = \boxed{33.0°}$

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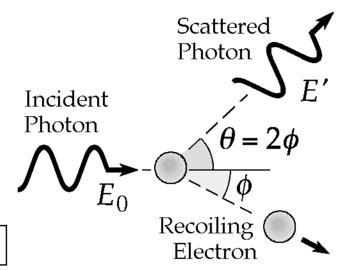

FIG. P40.28(a)

(b) From Equation [5], $\lambda' = \lambda_0\left(4\cos^2\phi - 1\right) = \lambda_0\left[4\left(\dfrac{1+x}{2+x}\right) - 1\right] = \lambda_0\left(\dfrac{2+3x}{2+x}\right)$.

Then, Equation [1] becomes

$$\frac{hc}{\lambda_0} = \frac{hc}{\lambda_0}\left(\frac{2+x}{2+3x}\right) + \gamma\, m_e c^2 - m_e c^2 \text{ or } \frac{E_0}{m_e c^2} - \frac{E_0}{m_e c^2}\left(\frac{2+x}{2+3x}\right) + 1 = \gamma.$$

Thus, $\gamma = 1 + x - x\left(\dfrac{2+x}{2+3x}\right)$, and with $x = 1.37$ we get $\gamma = 1.614$.

Therefore, $\dfrac{v}{c} = \sqrt{1-\gamma^{-2}} = \sqrt{1-0.384} = 0.785$ or $v = \boxed{0.785c}$.

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4 Nuclear astrophysics proposes that all the elements heavier than iron are formed in supernova explosions ending the lives of massive stars. Assume that at the time of the explosion the amounts of ^{235}U and ^{238}U were equal. How long ago did the star(s) explode that released the elements that formed our Earth? The present ^{235}U/^{238}U ratio is 0.007 25. The half-lives of ^{235}U and ^{238}U are 0.704 × 10^9 yr and 4.47 × 10^9 yr.

We have $N_{235} = N_{0,\,235}e^{-\lambda_{235}t}$ and $N_{238} = N_{0,\,238}e^{-\lambda_{238}t}$ $\dfrac{N_{235}}{N_{238}} = 0.007\,25 = e^{\left(-(\ln 2)t/T_{h,\,235}+(\ln 2)t/T_{h,\,238}\right)}$.

Taking logarithms,

$$-4.93 = \left(-\frac{\ln 2}{0.704\times 10^9\ \text{yr}} + \frac{\ln 2}{4.47\times 10^9\ \text{yr}}\right)t \ \text{o}\ -4.93 = \left(-\frac{1}{0.704\times 10^9\ \text{yr}} + \frac{1}{4.47\times 10^9\ \text{yr}}\right)(\ln 2)t$$

$$t = \frac{-4.93}{\left(-1.20\times 10^{-9}\ \text{yr}^{-1}\right)\ln 2} = \boxed{5.94\times 10^9\ \text{yr}}.$$

5 A star ending its life with a mass of two times the mass of the Sun is expected to collapse, combining its protons and electrons to form a neutron star. Such a star could be thought of as a gigantic atomic nucleus. If a star of mass 2 × 1.99 × 10^{30} kg collapsed into neutrons (m_n = 1.67 × 10^{-27} kg), what would its radius be? (Assume that $r = r_0A^{1/3}$.)

The number of nucleons in a star of two solar masses is $A = \dfrac{2\left(1.99\times 10^{30}\ \text{kg}\right)}{1.67\times 10^{-27}\ \text{kg/nucleon}} = 2.38\times 10^{57}$ nucleons.

Therefore: $r = r_0A^{1/3} = \left(1.20\times 10^{-15}\ \text{m}\right)\left(2.38\times 10^{57}\right)^{1/3} = \boxed{16.0\ \text{km}}$.

6A **T**he half-life of ^{131}I is 8.04 days. On a certain day, the activity of an iodine-131 sample is 6.40 mCi. What is its activity 40.2 days later?

$$R = R_0e^{-\lambda t} = (6.40\ \text{mCi})e^{-(\ln 2/8.04\ \text{d})(40.2\ \text{d})} = (6.40\ \text{mCi})\left(e^{-\ln 2}\right)^5 = (6.40\ \text{mCi})\left(\frac{1}{2^5}\right) = \boxed{0.200\ \text{mCi}}$$

6B Determine the activity of 1.00 g of ^{60}Co. The half-life of ^{60}Co is 5.27 yr.

$$R = \lambda N = \left(\frac{\ln 2}{5.27\ \text{yr}}\right)\left(\frac{1.00\ \text{g}}{59.93\ \text{g/mol}}\right)\left(6.02\times 10^{23}\right)\ R = \left(1.32\times 10^{21}\ \text{decays/yr}\right)\left(\frac{1\ \text{yr}}{3.16\times 10^7\ \text{s}}\right) = \boxed{4.18\times 10^{13}\ \text{Bq}}$$

7 A hydrogen atom is in its first excited state (n = 2). Using the Bohr theory of the atom, calculate
(a) the radius of the orbit,
(b) the linear momentum of the electron,
(c) the angular momentum of the electron,
(d) the kinetic energy of the electron,
(e) the potential energy of the system,
(f) the total energy of the system.
(h) **h**ow much energy is required to ionize hydrogen (a) when it is in the ground state? (b) when it is in the state for which n = 3?

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(a) $$r_2^2 = (0.052\,9\ \text{nm})(2)^2 = \boxed{0.212\ \text{nm}}$$

(b) $$m_e v_2 = \sqrt{\frac{m_e k_e e^2}{r_2}} = \sqrt{\frac{(9.11\times10^{-31}\ \text{kg})(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2)(1.60\times10^{-19}\ \text{C})^2}{0.212\times10^{-9}\ \text{m}}}$$

$$m_e v_2 = \boxed{9.95\times10^{-25}\ \text{kg}\cdot\text{m/s}}$$

(c) $$L_2 = m_e v_2 r_2 = (9.95\times10^{-25}\ \text{kg}\cdot\text{m/s})(0.212\times10^{-9}\ \text{m}) = \boxed{2.11\times10^{-34}\ \text{kg}\cdot\text{m}^2/\text{s}}$$

(d) $$K_2 = \frac{1}{2}m_e v_2^2 = \frac{(m_e v_2)^2}{2m_e} = \frac{(9.95\times10^{-25}\ \text{kg}\cdot\text{m/s})^2}{2(9.11\times10^{-31}\ \text{kg})} = 5.43\times10^{-19}\ \text{J} = \boxed{3.40\ \text{eV}}$$

(e) $$U_2 = -\frac{k_e e^2}{r_2} = -\frac{(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2)(1.60\times10^{-19}\ \text{C})^2}{0.212\times10^{-9}\ \text{m}} = -1.09\times10^{-18}\ \text{J} = \boxed{-6.80\ \text{eV}}$$

(f) $$E_2 = K_2 + U_2 = 3.40\ \text{eV} - 6.80\ \text{eV} = \boxed{-3.40\ \text{eV}}$$